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The large top quark mass expansion for Higgs boson decays into bottom quarks and into gluons

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Abstract

We calculate the large top quark mass expansions for the $H \to b\bar{b}$ decay rate in the order α_s^2 and for the $H \to$ gluons decay rate in the order α_s^3. The obtained expansions rapidly converge in the region of their validity, $M_H < 2m_t$, i.e. below the threshold of $t\bar{t}$ production.

1 Introduction

With the discovery of the top quark [1], the Higgs boson remains as the only fundamental particle of the Standard Model which is not found experimentally. Considerable efforts were devoted to calculate perturbative corrections within the Standard Model to different Higgs boson decay channels, for a review see [2]. In the present paper we calculate top quark mass corrections to Higgs boson decays into bottom quark-antiquark pairs and into gluons, i.e. to the Higgs boson partial widths $\Gamma(H \to b\bar{b})$ and $\Gamma(H \to$gluons$)$.

The decay process $H \to b\bar{b}$ is the dominant decay channel for the Higgs boson with an intermediate mass $M_H < 2M_W$ and will be of prime importance in the future experimental searches of the Higgs boson at colliders.

The decay channel $H \to$ gluons is interesting since heavy quarks that mediate this process contribute to the decay rate without being suppressed by their large mass which eventually may provide a way to count the number of heavy quarks beyond the Standard Model.

The purpose of the present paper is to examine the convergence properties of the series of top quark mass suppressed corrections to the Higgs boson decay rate in the region

$M_H < 2m_t$, where the large top quark mass expansion can be applied. These corrections a priori can be sizable but they turn out to be small for the processes considered so far. In ref. [3] we considered the top quark mass corrections to the Z boson and τ lepton decays into hadrons. In this letter we continue this program to the case of the Higgs decays into hadrons.

Throughout this paper in our calculations we use dimensional regularization [4] and the standard modification of the minimal subtraction scheme [5], the $\overline{MS}$ scheme [6].

2 The order α_s^2 corrections to $\Gamma(H \to b\bar{b})$

We use the optical theorem to calculate the top mass suppressed corrections to the partial decay width $\Gamma(H \to b\bar{b})$ in the order α_s^2. To this end we should calculate the imaginary parts of the 3-loop Higgs boson propagator diagrams of non-singlet and singlet types with top quark loops listed in fig.1a and fig.1b correspondingly.

Figure 1. Diagrams contributing to top mass suppressed corrections to the partial decay width $\Gamma(H \to b\bar{b})$. Thin lines indicate bottom quark propagators, thick lines indicate top quark propagators and spiral lines indicate gluons. The symbol $\otimes$ indicates a Yukawa type Higgs–quark vertex. Diagrams 1a are of the non-singlet type, Diagram 1b is of the singlet type.

To extract the contribution to this channel from the singlet "double triangle" diagram in fig.1b we should subtract from the imaginary part of this diagram the contribution to $\Gamma(H \to \text{gluons})$ associated with two gluon cut in this singlet diagram. The contribution from this two gluon cut can be straightforwardly calculated [8] by the use of Feynman parameters.

For the calculation of the propagator diagrams in fig.1 we applied the diagrammatic large top quark mass expansion to each of the diagrams separately, using the technique which was previously used in [3] and which is based on the method developed in [7]. Although the formal parameter of this expansion is not small (the mass ratio M_H/m_t can be larger than 1) this expansion is applicable below the threshold for the production of $t\bar{t}$ pairs, i.e. $M_H < 2m_t$. (One can say that the real expansion parameter is $M_H/2m_t$.) For the actual calculations we relied on the symbolic manipulation program FORM[9] and

we used the package MINCER[10] together with additional FORM routines to perform massive integrals.

The Yukawa type Higgs–quark couplings contribute a factor m_b^2 for the non-singlet diagrams and a factor $m_b m_t$ for the singlet diagram. We work in the leading order in small b-quark mass, $m_b << M_H$, and nullify other light quark masses. This technically means that for the non-singlet diagrams the mass in the b-quark propagators is nullified from the start but for the singlet diagram one power of the b-quark mass from the propagators must be kept to prevent the trace over the Dirac matrices in the b-triangle from vanishing. The higher small m_b^2-corrections can be found in [11] for non-singlet contributions and in [12] for the full case.

After addition of the massless (m_t independent) non-singlet contributions calculated in [13] we obtain the following result for the large top quark mass expansion of the Higgs decay rate into $b\bar{b}$ in the leading order of the b-quark mass

$$\Gamma(H \to b\bar{b}) = \Gamma^{NS}(H \to b\bar{b}) + \Gamma^{S}(H \to b\bar{b}),$$

$$\Gamma^{NS}(H \to b\bar{b}) = \frac{G_F M_H (m_b^{(6)})^2}{4\pi\sqrt{2}} n \left[1 + \left(\frac{\alpha_s^{(6)}}{\pi}\right) f_1^{NS,0} + \left(\frac{\alpha_s^{(6)}}{\pi}\right)^2 \left(f_2^{NS,0} + f_2^{NS,1}\frac{M_H^2}{m_t^2} + f_2^{NS,2}\frac{M_H^4}{m_t^4} + f_2^{NS,3}\frac{M_H^6}{m_t^6} + O(\frac{M_H^8}{m_t^8})\right)\right], \quad (1)$$

$$f_1^{NS,0} = C_F(\tfrac{17}{4}),$$

$$\begin{aligned} f_2^{NS,0} = {} & C_F^2 \left[\tfrac{691}{64} - \tfrac{9}{4}\zeta_3 - \tfrac{3}{8}\pi^2 - \tfrac{105}{16}\ln(\tfrac{M_H^2}{\mu^2}) + \tfrac{9}{8}\ln^2(\tfrac{M_H^2}{\mu^2})\right] \\ & + C_A C_F \left[\tfrac{893}{64} - \tfrac{31}{8}\zeta_3 - \tfrac{11}{48}\pi^2 - \tfrac{71}{12}\ln(\tfrac{M_H^2}{\mu^2}) + \tfrac{11}{16}\ln^2(\tfrac{M_H^2}{\mu^2})\right] \\ & + T_F C_F N_f \left[-\tfrac{65}{16} + \zeta_3 + \tfrac{1}{12}\pi^2 + \tfrac{11}{6}\ln(\tfrac{M_H^2}{\mu^2}) - \tfrac{1}{4}\ln^2(\tfrac{M_H^2}{\mu^2})\right] \\ & + T_F C_F \left[\tfrac{337}{72} - \zeta_3 - \tfrac{1}{12}\pi^2 - \tfrac{11}{6}\ln(\tfrac{M_H}{m_t^2}) + \tfrac{1}{4}\ln^2(\tfrac{M_H^2}{m_t^2})\right], \end{aligned}$$

$$f_2^{NS,1} = T_F C_F \left[\tfrac{107}{450} - \tfrac{1}{15}\ln(\tfrac{M_H^2}{m_t^2})\right],$$

$$f_2^{NS,2} = T_F C_F \left[-\tfrac{529}{58800} + \tfrac{1}{280}\ln(\tfrac{M_H^2}{m_t^2})\right],$$

$$f_2^{NS,3} = T_F C_F \left[\tfrac{2719}{3572100} - \tfrac{1}{2835}\ln(\tfrac{M_H^2}{m_t^2})\right],$$

$$\Gamma^S(H \to b\bar{b}) = \frac{G_F M_H (m_b^{(6)})^2}{4\pi\sqrt{2}} n \left[\left(\frac{\alpha_s^{(6)}}{\pi} \right)^2 \left(f_2^{S,0} + f_2^{S,1} \frac{M_H^2}{m_t^2} + f_2^{S,2} \frac{M_H^4}{m_t^4} + f_2^{S,3} \frac{M_H^6}{m_t^6} + O(\frac{M_H^8}{m_t^8}) \right) \right], \tag{2}$$

$$f_2^{S,0} = T_F C_F \left[-\ln(\tfrac{M_H^2}{m_t^2}) + \tfrac{14}{3} + \left\{ -\tfrac{1}{6}\pi^2 - \tfrac{2}{3} + \tfrac{1}{6} \ln^2(\tfrac{m_b^2}{M_H^2}) \right\} \right],$$

$$f_2^{S,1} = T_F C_F \left[-\tfrac{41}{1080} \ln(\tfrac{M_H^2}{m_t^2}) + \tfrac{2011}{16200} + \left\{ -\tfrac{7}{720}\pi^2 - \tfrac{7}{180} + \tfrac{7}{720} \ln^2(\tfrac{m_b^2}{M_H^2}) \right\} \right],$$

$$f_2^{S,2} = T_F C_F \left[-\tfrac{47}{15120} \ln(\tfrac{M_H^2}{m_t^2}) + \tfrac{28307}{3175200} + \left\{ -\tfrac{1}{1008}\pi^2 - \tfrac{1}{252} + \tfrac{1}{1008} \ln^2(\tfrac{m_b^2}{M_H^2}) \right\} \right],$$

$$f_2^{S,3} = T_F C_F \left[-\tfrac{59}{168000} \ln(\tfrac{M_H^2}{m_t^2}) + \tfrac{100381}{105840000} + \left\{ -\tfrac{13}{100800}\pi^2 - \tfrac{13}{25200} + \tfrac{13}{100800} \ln^2(\tfrac{m_b^2}{M_H^2}) \right\} \right],$$

where we presented separately the contributions from the non-singlet diagrams, $\Gamma^{NS}(H \to b\bar{b})$, and from the singlet diagram, $\Gamma^S(H \to b\bar{b})$. The contribution of the two gluon cut is subtracted from the singlet part (the corresponding terms are indicated in curly brackets). G_F is the Fermi constant. $C_F = \frac{4}{3}$ and $C_A = 3$ are the Casimir operators of the fundamental and adjoint representation of the colour group $SU(n)$, n = 3 is the number of quark colours, $T_F = \frac{1}{2}$ is the trace normalization of the fundamental representation. The result is for 6-flavour QCD, $N_f = 6$, $\alpha_s^{(6)}$ is the strong coupling constant and $m_b^{(6)}(\mu)$, $m_t(\mu)$ are running $\overline{MS}$ bottom and top quark masses. The effects of using the pole m_b mass instead of the running m_b mass can be found in [14].

The singlet type coefficients $f_2^{S,0}$ and $f_2^{S,1}$ agree with the previous calculations [15] and [12] correspondingly. The non-singlet type coefficients agree with the previous calculation [16]. The singlet coefficients $f_2^{S,2}$, $f_2^{S,3}$ are new results.

To obtain the result in effective 5-flavour QCD where the non-singlet top quark contributions decouple [17], one should substitute the b-quark mass $m_b^{(6)}$ and the coupling constant $\alpha_s^{(6)}$ in terms of their values in the effective 5-flavour QCD by appling the known decoupling relations [18, 19, 3] :

$$\frac{\alpha_s^{(6)}(\mu)}{\pi} = \frac{\alpha_s^{(5)}(\mu)}{\pi} + \left(\frac{\alpha_s^{(5)}(\mu)}{\pi} \right)^2 \frac{T_F}{3} \ln(\frac{\mu^2}{m_t^2(\mu)}) + O(\alpha_s^3) \tag{3}$$

$$m_b^{(6)}(\mu) = m_b^{(5)}(\mu) \left[1 + \left(\frac{\alpha_s^{(5)}(\mu)}{\pi} \right)^2 T_F C_F \left(-\frac{1}{8} \ln^2(\frac{\mu^2}{m_t^2(\mu)}) + \frac{5}{24} \ln(\frac{\mu^2}{m_t^2(\mu)}) - \frac{89}{288} \right) + O(\alpha_s^3) \right] \tag{4}$$

Substitution of (3) and (4) in (1)+(2) and putting $\mu = M_H$ gives the following result for

the Higgs decay rate into $b\bar{b}$ in effective 5-flavour QCD

$$\Gamma(H \to b\bar{b}) = \frac{3G_F M_H (m_b^5)^2}{4\pi\sqrt{2}} \left[1 + \left(\frac{\alpha_s^{(5)}}{\pi}\right) f_1 + \left(\frac{\alpha_s^{(5)}}{\pi}\right)^2 f_2 + O\left(\frac{\alpha_s^{(5)}}{\pi}\right)^3\right], \tag{5}$$

$$f_1 = \tfrac{17}{3} \approx 5.66667,$$

$$\begin{aligned}
f_2 = {} & \tfrac{9235}{144} - \tfrac{97}{6}\zeta_3 - \tfrac{17}{12}\pi^2 - \tfrac{2}{3}\ln(x) + \tfrac{1}{9}\ln^2(y) \\
& + \left[\tfrac{5233}{24300} - \tfrac{7}{1080}\pi^2 - \tfrac{113}{1620}\ln(x) + \tfrac{7}{1080}\ln^2(y)\right] x \\
& + \left[-\tfrac{1837}{680400} - \tfrac{1}{1512}\pi^2 + \tfrac{1}{3240}\ln(x) + \tfrac{1}{1512}\ln^2(y)\right] x^2 \\
& + \left[\tfrac{3411287}{4286520000} - \tfrac{13}{151200}\pi^2 - \tfrac{3193}{6804000}\ln(x) + \tfrac{13}{151200}\ln^2(y)\right] x^3 + O(x^4) \\
& \approx 30.71675 - 0.66667\ln(x) + 0.11111\ln^2(y) \\
& + \left[0.15138 - 0.06975\ln(x) + 0.0064815\ln^2(y)\right] x \\
& + \left[-0.009227 + 0.00030864\ln(x) + 0.00066137\ln^2(y)\right] x^2 \\
& + \left[-0.00005276 - 0.0004693\ln(x) + 0.00008598\ln^2(y)\right] x^3 + O(x^4)
\end{aligned}$$

where, $x = M_H^2/m_t^2$, $y = m_b^2/M_H^2$.

Please note in the leading order of the large top quark mass expansion the presence of $\ln(M_H^2/m_t^2)$ coming from the singlet contribution which is not affected by the decoupling procedure (since the singlet type contributions are renormalized independently of the non-singlet ones). It shows the violation of decoupling for the considered process due to the singlet contribution, which was first found in [15, 12].

We conclude this section with stressing the fast convergence of the obtained large top quark mass expansion for $\Gamma(H \to b\bar{b})$ in the region below the threshold of $t\bar{t}$ production, $M_H < 2m_t$ as can be seen from the fast decrease of the coefficients of the expansion. Even in the region close to this threshold (where $x \approx 4$) one observes a fast convergence of the large top quark mass expansion.

3 The order α_s^3 corrections to $\Gamma(H \to \text{gluons})$

Higgs boson decay into gluons is possible since both the Higgs particle and gluons couple to massive quark loops. An important property of this partial decay rate is that contributions from heavy quarks in these loops are not suppressed by their mass such that measurement of this decay rate counts the number of heavy quarks. The contributions to $\Gamma(H \to \text{gluons})$ start from the order α_s^2 (and this process is therefore not as prominent phenomenologically as the Higgs decay rate into bottom quarks which receives tree level contributions).

$\Gamma(H \to \text{gluons})$ is known[20, 21] in the order α_s^3 in the leading order of the large top quark mass expansion and the α_s^3 correction turns out to be large (about 2/3 of the leading α_s^2 contribution).

In this section we present the higher orders in the large top quark mass expansion of the Higgs decay rate into (two and three) gluons $H \to gg(g)$ mediated by top quark loops. The $gq\overline{q}$ final states (with light quarks q=u,d,s,c,b) generated in the chain $H \to gg \to gq\overline{q}$ are also included. For this process we work in the approximation in which the first five quark flavours are massless.

By applying the optical theorem, the necessary contributions are given by the imaginary part of the singlet 4-loop diagrams of the Higgs propagator in which each of the external Higgs vertices is located in a different top quark loop. Clearly, for a Higgs mass below the top threshold, $M_H < 2m_t$, the only physical cuts that can be drawn in these propagator type diagrams are through gluons and light quarks.

All contributing diagrams are of the singlet type and were encountered previously in the calculation of $\Gamma(Z \to \text{hadrons})$ [3] where a complete list of the necessary 4-loop singlet diagrams was given. Applying the diagrammatic large quark mass expansion to all contributing diagrams yields the result in six flavour QCD, $N_f = 6$,

$$\Gamma(H \to \text{gluons}) = \frac{G_F M_H^3}{72\pi\sqrt{2}} \left(\frac{\alpha_s^{(6)}}{\pi}\right)^2 T_F^2 D \left\{ 1 + h_2^1 \frac{M_H^2}{m_t^2} + h_2^2 \frac{M_H^4}{m_t^4} + O\left(\frac{M_H^6}{m_t^6}\right) \right.$$
$$\left. + \left(\frac{\alpha_s^{(6)}}{\pi}\right) \left[h_3^0 + h_3^1 \frac{M_H^2}{m_t^2} + h_3^2 \frac{M_H^4}{m_t^4} + O\left(\frac{M_H^6}{m_t^6}\right) \right] \right\}, \tag{6}$$

$$h_2^1 = \left(\tfrac{7}{60}\right),$$

$$h_2^2 = \left(\tfrac{1543}{100800}\right),$$

$$h_3^0 = C_A \left[\tfrac{103}{12} - \tfrac{11}{6} \ln(\tfrac{M_H^2}{\mu^2})\right] + C_F \left(-\tfrac{3}{2}\right)$$
$$+T_F N_f \left[-\tfrac{7}{3} + \tfrac{2}{3} \ln(\tfrac{M_H^2}{\mu^2})\right] + T_F \left[\tfrac{7}{3} - \tfrac{2}{3} \ln(\tfrac{M_H^2}{m_t^2})\right],$$

$$h_3^1 = C_A \left[\tfrac{71}{80} - \tfrac{77}{360} \ln(\tfrac{M_H^2}{\mu^2})\right] + C_F \left[\tfrac{13}{720} - \tfrac{7}{40} \ln(\tfrac{\mu^2}{m_t^2})\right]$$
$$+T_F N_f \left[-\tfrac{29}{120} + \tfrac{7}{90} \ln(\tfrac{M_H^2}{\mu^2})\right] + T_F \left[\tfrac{29}{120} - \tfrac{7}{90} \ln(\tfrac{M_H^2}{m_t^2})\right],$$

$$h_3^2 = C_A \left[\tfrac{47459}{432000} - \tfrac{16973}{604800} \ln(\tfrac{M_H^2}{\mu^2})\right] + C_F \left[\tfrac{83}{10080} - \tfrac{1543}{33600} \ln(\tfrac{\mu^2}{m_t^2})\right]$$
$$+T_F N_f \left[-\tfrac{89533}{3024000} + \tfrac{1543}{151200} \ln(\tfrac{M_H^2}{\mu^2})\right] + T_F \left[\tfrac{89533}{3024000} - \tfrac{1543}{151200} \ln(\tfrac{M_H^2}{m_t^2})\right],$$

where $D = n^2 - 1$ is the number of generators of the colour group $SU(n)$ ($D = 8$ for

QCD). In the leading order of the large top quark mass expansion our α_s^3 result agrees with [21]. Explicit checks show that the coefficients of the logarithms in eq. (6) are in agreement with the required renormalization group invariance of the physical quantity $\Gamma(H \to \text{gluons})$. Furthermore, the results that are presented in this paper were obtained in an arbitrary covariant gauge for the gluon fields i.e. keeping the gauge parameter as a free parameter in the calculations. The explicit cancellation of the gauge dependence in the physical quantities gives a good check of the results.

Applying the decoupling relation (3), putting $\mu = M_H$ and substituting the QCD colour factors gives the following result for effective 5 flavour QCD

$$\Gamma(H \to \text{gluons}) = \frac{G_F M_H^3}{36\pi\sqrt{2}} \left(\frac{\alpha_s^{(5)}}{\pi}\right)^2 \left[h_2 + \left(\frac{\alpha_s^{(5)}}{\pi}\right) h_3 + O(\alpha_s)^2\right], \tag{7}$$

$$\begin{aligned} h_2 &= 1 + \tfrac{7}{60}x + \tfrac{1543}{100800}x^2 + O(x^3) \\ &\approx 1 + 0.116667x + 0.015308x^2 + O(x^3), \end{aligned}$$

$$\begin{aligned} h_3 &= \tfrac{215}{12} + \left[\tfrac{2249}{1080} - \tfrac{7}{30}\ln(x)\right] x + \left[\tfrac{1612013}{6048000} - \tfrac{1543}{25200}\ln(x)\right] x^2 + O(x^3) \\ &\approx 17.9167 + [2.08241 - 0.23333\ln(x)]\, x + [0.26654 - 0.06123\ln(x)]\, x^2 + O(x^3). \end{aligned}$$

where $x = M_H^2/m_t^2$.

For completeness we will also give the total hadronic decay rate of the Higgs boson (in the leading order of the small b-quark mass expansion). This result is obtained by summing the partial decay rates $\Gamma(H \to b\bar{b})$ and $\Gamma(H \to \text{gluons})$ i.e. by applying the decoupling relations to the sum of eqs. (1), (2) and (6) without subtracting the two gluon cut in eq.(2).

$$\begin{aligned} &\Gamma_{\text{tot}}(H \to \text{hadrons}) = \\ &\frac{3G_F M_H (m_b^{(5)})^2}{4\pi\sqrt{2}} \left[1 + \left(\frac{\alpha_s^{(5)}}{\pi}\right)\frac{17}{3} + \left(\frac{\alpha_s^{(5)}}{\pi}\right)^2 \left\{\frac{9299}{144} - \frac{97}{6}\zeta_3 - \frac{47}{36}\pi^2 - \frac{2}{3}\ln(x)\right.\right. \\ &\quad \left.\left. + \left[\frac{5863}{24300} - \frac{113}{1620}\ln(x)\right] x + \left[-\frac{37}{680400} + \frac{1}{3240}\ln(x)\right] x^2 + O(x^3)\right\} + O(\alpha_s^3) + O(m_b^2)\right] \\ &+ \frac{G_F M_H^3}{36\pi\sqrt{2}} \left[\left(\frac{\alpha_s^{(5)}}{\pi}\right)^2 \left[1 + \frac{7}{60}x + \frac{1543}{100800}x^2 + O(x^3)\right] + \left(\frac{\alpha_s^{(5)}}{\pi}\right)^3 \left\{\frac{215}{12}\right.\right. \\ &\quad \left.\left. + \left[\frac{2249}{1080} - \frac{7}{30}\ln(x)\right] x + \left[\frac{1612013}{6048000} - \frac{1543}{25200}\ln(x)\right] x^2 + O(x^3)\right\} + O(\alpha_s^4) + O(m_b^2)\right] \end{aligned}$$

$$\approx \frac{3G_F M_H (m_b^{(5)})^2}{4\pi\sqrt{2}} \left[1 + \left(\frac{\alpha_s^{(5)}}{\pi}\right) 5.66667 + \left(\frac{\alpha_s^{(5)}}{\pi}\right)^2 \left\{32.2578 - 0.6667\ln(x) + \left[0.2413\right.\right.\right.$$

$$\left.\left.\left. -0.06975\ln(x)\right] x + \left[-0.00005438 + 0.0003086\ln(x)\right] x^2 + O(x^3)\right\} + O(\alpha_s^3) + O(m_b^2)\right]$$

$$+\frac{G_F M_H^3}{36\pi\sqrt{2}} \left[\left(\frac{\alpha_s^{(5)}}{\pi}\right)^2 \left[1 + 0.11667x + 0.01531x^2 + O(x^3)\right] + \left(\frac{\alpha_s^{(5)}}{\pi}\right)^3 \left\{17.9167\right.\right.$$

$$\left.\left. +\left[2.0824 - 0.2333\ln(x)\right] x + \left[0.2665 - 0.06123\ln(x)\right] x^2 + O(x^3)\right\} + O(\alpha_s^4) + O(m_b^2)\right],$$

where $x = M_H^2/m_t^2$. Here the terms proportional to m_b^2 originate from $\Gamma(H \to b\bar{b})$ and the terms proportional to M_H^3 come from $\Gamma(H \to \text{gluons})$ (both in the leading order of the small m_b-expansion).

We conclude that the large top quark mass expansion converges rapidly for $\Gamma(H \to b\bar{b})$, $\Gamma(H \to \text{gluons})$ and $\Gamma_{\text{tot}}(H \to \text{hadrons})$ in the region of its applicability $M_H < 2m_t$.

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